AAEC 6305 - Optimization and Machine Learning

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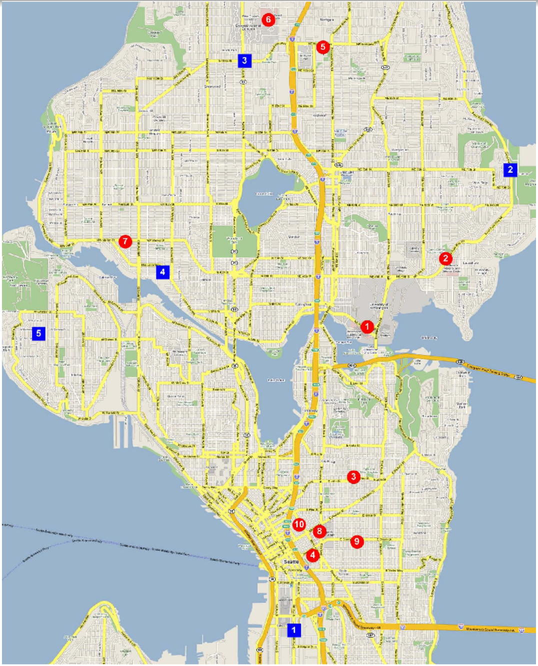
Introduction

The paper

Stochastic optimization of medical supply location and distribution in disaster management (Meta & Zabinsky, 2010)

- Logistics of humanitarian aid during disasters
- Stage one: minimize cost of warehouse operation and expected second stage solution with respect to disaster scenarios
- Stage two: minimize total transportation duration and penalty of unfulfilled demand
- Mixed-integer programming (transportation plan): minimize total transportation time of assigned vehicles

Map of Seattle



Overview

Set of all medical supplies = K



Set of all warehouses = I



Variables for stage one

- I represents the set of warehouses
- K represents the set of types of medical supplies
- s_{ik} represents the decision variable for the inventory level of medical supply k in warehouse i for all $i \in I$ and $k \in K$
- x_i represents a binary variable
  - 1 if the warehouse i is selected to be operating
  - 0 otherwise; for each warehouse $i \in I$
- g_i represents the warehouse operating costs
- e_k represents the maximum amount available of each medical supply type
- l_{ik} represents the storage capacity of each warehouse for each medical supply type for all $i \in I$ and $k \in K$

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Stage one model

Warehouse selection and inventory decisions

$$\min_{s} \sum_{i \in I} g_i x_i + E_{\Xi}[Q(x, s, \xi)] \tag{1}$$

subject to

$$\sum_{i \in I} s_{ik} \leq e_k; \forall\ k \in K \tag{2}$$

$$s_{ik} \leq l_{ik} x_i; \forall\ i \in I, k \in K \tag{3}$$

$$x_i \in \{0, 1\}, s_{ik} \geq 0; \forall\ i \in I, k \in K \tag{4}$$

Variables for stage two

- J represents the set of hospitals in addition to those used in the first stage
- $t_{ijk}(\xi)$ represents the recourse decision variable, which is the amount of medical supply k to be delivered from warehouse i to hopsital j under disaster scenario ξ
- $c_{ij}(\xi)$ represents the transportation time between warehouse i to hospital j
- $w_{jk}(\xi)$ represents unfulfilled demand at hospital j of medical supply type k under scenario ξ
- $y_{jk}(\xi)$ represents the amount of unfulfilled demand
- $d_{jk}(\xi)$ represents the demand of medical supply type k at hospital j under scenario ξ
- τ_{jk} represents the upper limit for penalty of unsatisfied demands for each hospital j and medical supply type k

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Stage two model

Transportation plans and demand satisfaction decisions

$$Q(x, s, \xi) = \min \sum_{i \in I} \sum_{j \in J} \Big(c_{ij}(\xi) \sum_{k \in K} t_{ijk}(\xi) \Big) + \sum_{j \in J} \sum_{k \in K} w_{jk}(\xi) y_{jk}(\xi) \tag{5}$$

subject to

$$\sum_{j} t_{ijk}(\xi) \leq s_{ik}; \forall\ i \in I, k \in K \tag{6}$$

$$\sum_{i} t_{ijk}(\xi) = d_{jk}(\xi) - y_{jk}(\xi); \forall\ j \in J, k \in K \tag{7}$$

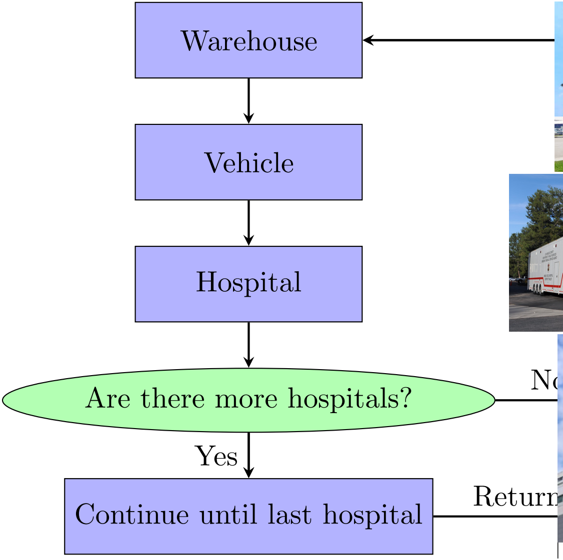
$$w_{jk}(\xi) y_{jk}(\xi) \leq \tau_{jk}; \forall\ j \in J, k \in K \tag{8}$$

$$t_{ijk}(\xi), y_{jk}(\xi) \geq 0; \forall\ i \in I, j \in J, k \in K \tag{9}$$

Transportation plan

Mixed-integer programming (MIP) model explanation

- Subproblem of stage two
- Dispatches vehicles from warehouses to hospitals under each scenario ξ
- Uses predetermined routes at the expense of preprocessing effort
- There are already daily transportation plans from warehouses to hospitals
- Expands alternative routes to avoid bridges and highways that are vulnerable to certain disasters (i.e. earthquakes)
- *Route* is defined as an ordered list of subset hospitals from an initial warehouse


Warehouse
Vehicle
Hospital
Are there more hospitals?
Yes
No
Continue until last hospital
Return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South Entrance
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Variables for MIP

- V represents the set of available vehicles
- R represents the set of possible routes
- z_{vr} represents a binary variable
- m_{ijkvr} represents the decision variable of the transportation amount of k-type medical supply along route r by vehicle v from warehouse i to hospital j
- q_r represents the travel time along route r
  - Note: q_r is not the sum of $c_{ij}(\xi)$ because the route r may include several hospitals whereas $c_{ij}(\xi)$ includes exactly one hospital
- l represents a set of disjoint supply types
  - $l = 1$ for types that require refrigeration
  - $l = 2$ for ones that do not
- h_{vl} represents the carrying capacity of vehicle v with the classification of refrigeration l

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MIP model

Transportation plans

$$\min_{m} \sum_{r \in R} q_r \Big(\sum_{v \in V} z_{vr} \Big) \tag{10}$$

subject to

$$\sum_{i \in I} \sum_{j \in J} \sum_{k \in K_l} m_{ijkvr} \leq h_{vl} z_{vr}; \forall\ v \in V, r \in R, i \in \{1, 2\} \tag{11}$$

$$\sum_{v \in V} \sum_{r \in R} m_{ijkvr} = t_{ijk}(\xi); \forall\ i \in I, j \in J, k \in K \tag{12}$$

$$\sum_{r \in R} z_{vr} \leq 1; \forall\ v \in V \tag{13}$$

$$m_{ijkvr} \leq 0; \forall\ i \in I, j \in J, k \in K, v \in V, r \notin R_{ij} \tag{14}$$

$$z_{vr} \in \{0, 1\}, m_{ijkvr} \geq 0; \forall\ i \in I, j \in J, k \in K, v \in V, r \in R \tag{15}$$

Case study: potential earthquakes in Seattle

Earthquakes

- Seattle fault (6.7 magnitude)
  - Assumption: damage southern part of the city and I-5
  - Probability: 0.4
- Cascadia fault (9.0 magnitude)
  - Assumption: damage north part of the city and smaller bridges
  - Probability: 0.6

Time of day

- Working hours (W)
- Rush hours (R)
- Nonworking hours (N)
- Mon-Sat: 8 W, 5 R, 11 N; Sun: All N

Tables

Probabilities of scenarios

Scenario	Seattle fault			Cascadia fault		
	W	R	N	W	R	N
Probability	0.11	0.07	0.22	0.17	0.11	0.32

Warehouse capacities and operating costs

Warehouse	Capacity ($\$10^3$ units)	Cost ($\$10^6$)	Cost/capacity ($\$10^3$/unit)
1	20	25	1.25
2	25	20	0.80
3	30	12	0.40
4	10	6	0.60
5	5	12	2.40

Transportation table

Warehouse	Hospital	Seattle fault			Cascadia fault		
		W	R	N	W	R	N
1	1	77	210	44	44	90	11
1	2	105	210	60	60	90	15
1	3	27	27	18	18	18	9
1	4	15	15	10	10	10	5
1	5	105	210	60	60	90	15
1	6	112	210	64	64	90	16
1	7	147	245	84	84	105	21
1	8	18	18	12	12	12	6
1	9	24	24	16	16	16	8
1	10	18	18	12	12	12	6
1	1–2	89	222	50	62	108	23
1	9–3	42	42	28	28	28	14
1	1–5–6	252	469	144	144	201	36
1	4–8–10	30	30	20	20	20	10
1	4–10–3	45	45	30	30	30	15
1	5–6–7	151	256	83	129	159	61
1	7–2–1	211	329	116	180	231	85
1	4–8–10–9–3	63	63	42	42	42	21
2	1	25	25	11	39	39	25
2	2	14	14	7	21	21	14
2	3	133	133	76	76	57	19
2	4	126	245	72	72	105	18
2	5	26	26	13	39	39	26
2	6	32	50	16	48	75	32
2	7	42	60	21	63	90	42
2	8	133	245	76	76	105	19
2	9	140	245	80	80	105	20
2	10	119	245	68	68	105	17
2	2–1	28	28	14	42	42	28
2	5–6	38	38	19	57	57	38
2	10-4	128	254	74	74	111	20
2	7–6–5	102	138	51	153	207	102
2	2–1–10–4	238	448	134	162	222	58
2	4–9–8–10	153	272	90	90	123	27
2	2–1–3–9–8–4–10	533	943	282	406	577	155
2	2–1–10–4–8–9–3	533	943	282	406	577	155
3	1	98	245	56	56	105	14
3	2	112	175	64	64	75	16
3	3	112	245	64	64	105	16
3	4	98	245	56	56	105	14
3	5	14	14	7	21	21	14
3	6	8	8	4	12	12	8
3	7	24	24	12	36	36	24
3	8	45	105	30	30	70	15
3	9	51	105	34	34	70	17
3	10	15	15	10	10	10	5
3	2–1	126	189	71	85	96	30
3	6–5	18	18	9	27	27	18
3	10–4	24	24	16	16	16	8
3	3–1–2	216	380	90	246	420	120
3	6–5–7	46	46	23	69	69	46
3	7–1–2	88	108	44	132	162	88
3	2–1–3–9	309	474	155	265	361	111
3	4–9–8–10	125	272	74	74	123	23
4	1	24	24	12	36	36	24
4	2	34	50	17	51	75	34
4	3	119	119	68	68	51	17
4	4	119	119	68	68	51	17
4	5	34	34	17	51	51	34
4	6	30	50	15	45	75	30
4	7	40	70	20	60	105	40
4	8	54	90	36	36	60	18
4	9	57	105	38	38	70	19
4	10	51	51	34	34	34	17
4	1–2	36	36	18	54	54	36
4	3–9	137	137	80	80	63	23
4	5–6	46	46	23	69	69	46
4	7–6	90	134	46	135	204	90
4	7–6–5	100	148	50	150	222	100
4	1–2–5–6	211	295	118	154	165	61
4	4–9–8–10	146	146	86	86	69	26
4	10–4–9–8–3	102	102	68	68	68	34
5	1	147	210	84	84	90	21

Table 5 (*continued*)

Warehouse	Hospital	Seattle fault			Cascadia fault	
		W	R	N	W	R
5	2	56	56	28	84	84
5	3	154	154	88	88	66
5	4	66	66	44	44	44
5	5	108	81	27	189	189
5	6	96	72	24	168	168
5	7	48	36	12	84	84
5	8	69	69	46	46	46
5	9	75	75	50	50	50
5	10	63	90	42	42	60
5	1–2	159	222	90	102	108
5	3–9	172	172	100	100	78
5	5–6	120	93	33	207	207
5	7–2–1	112	120	44	180	210
5	7–5–6	108	114	42	174	201
5	1–2–5–6	334	481	190	202	219
5	10–8–9–4	96	123	64	64	82
5	10–4–8–9–3	114	141	76	76	94

Warehouse	Hospital	Seattle fault			Cascadia fault	
		W	R	N	W	R
1	3	4969	3732	6466	5922	5047
	4		3454	4254	5422	3917
	8	1874	4617	4213	1872	
	9	5723	3686	1773	6784	4036
	9–3					
	4–8–10	1532–5468-0				3197–3803-0
	Total	**19,566**	**15,489**	**16,706**	**20,000**	**20,000**
2	1	6313	6042	6485	7000	5264
	1				2234	
	2	3409	3857		5296	
	3					100
	3					
	5				6706	
	2–1			3994–3006		3958-3042
	Total	**9722**	**9899**	**13,485**	**21,236**	**12,364**
3	1					
	5			4836		3564
	6			2913		
	7	7000	5932	3869	7000	6830
	7	3021			5410	
	8				6310	
	10	5214	3498	2189		3006
	6–5	3129–2293	2508–3487		3801–479	1814–5186
	6–5–7					
	4–9–8–10				0–0–952–6048	
	Total	**20,657**	**15,425**	**13,807**	**30,000**	**20,400**

More tables

Demand amounts of hospitals

Hospital	Seattle fault			Cascadia fault		
	W	R	N	W	R	N
1	6313	6042	9491	9234	8306	13,624
2	3409	3857	3994	5296	3958	7149
3	4969	3732	6466	5922	5147	9357
4	1532	3454	4254	5422	7114	7507
5	2293	3487	4836	7185	8750	10,258
6	3129	2508	2913	3801	1814	2112
7	10,021	5932	3869	12,410	6830	7639
8	7342	4617	4213	9134	3803	5924
9	5723	3686	1773	6784	4036	4382
10	5214	3498	2189	6048	3006	3861

Last table

Transport time coefficients

Path type	Seattle fault			Cascadia fault		
	W	R	N	W	R	N
Paths through I-5	7	7	4	4	3	1
Paths through small bridges	4	3	1	7	7	4
North paths	2	2	1	3	3	2
South paths	3	3	2	2	2	1

Concluding remarks

Practical implementations

- Applicable to a variety disaster scenarios in other cities
- Claims up to 10 medical supply types can be solved in a reasonable time during disaster
- Used in planners and first responder training, such as the simulation and visualization environment RimSim developed for Seattle

Required information

- Availability of medical supplies
- Warehouse locations
- Pre-disaster capacities and operating costs
- Frequently used routes and alternative routes according to infrastructure damage